Macrobend sensor via the use of a hollow-core splice fiber: theory and experiments

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Abstract. The characteristics of the use of a hollow-core splice scheme for macrobend measurements are discussed both theoretically and experimentally. A perturbation theory for the modes of a bent hollow-core fiber and its loss characterization are developed so as to better understand the characteristics of the scheme. The maximal detection range of fabricated sensors with the proposed scheme is experimentally determined to be as large as a few hundred millimeters relative to the radius of curvature. In addition, the numerical estimation of the loss characteristics using the scheme shows modal trends which are in good agreement with experimental data. © 2002 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.1487866]

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1 Introduction

Sensing physical parameters by means of fiber-optic devices is an important ongoing issue in optical engineering and has been the subject of considerable recent attention.1-15 Fiber-optic sensors utilizing bend effects on a fiber have been explored to measure bends. Thus, the mode and loss characteristics of bent fibers have been investigated intensively, and their applications to bend-type sensors have been reported.4-6,16-22 In general, bend-type sensors using single-mode fibers (SMFs) are candidates for use in microbend measurements, and their characteristics have been well analyzed both theoretically and experimentally.3,5,16-22 Typically in the case of a common SMF, the detectable radius of curvature of a bent fiber is as small as a few millimeters. Thus, a special fiber, such as a heterocore splice fiber has been investigated for use in macrobend measurements, in which the bend-loss sensitivity can be enhanced to tens of millimeters relative to the radius of curvature.6 Elliptical-core fibers have been investigated as macrobend sensors using bending-induced phase shift, which showed a good linearity for a bend-angle increment. However, the elliptical-core fiber should be as long as a few centimeters to obtain sufficient sensitivity.8-12 In fact, the sensitivity is dependent on the birefringence of the elliptical-core fiber and its geometry. Long-period fiber gratings have also been investigated by taking advantage of a spectral-response change based on bend curvatures.13-15 The spectral characteristics of bent long-period fiber gratings have been reported and proposed for bend-sensing applications. It was shown that very small curvatures, less than 1 m-1, could be detected; however, the spectral response to variations in the environmental temperature need to be resolved and, in addition, the interrogation of the spectral-response change is not simple, as compared to conventional bend-loss-type sensors. Even though they have some specific restrictions based on individual cases, they represent a promising approach in the macrobend sensor area.

Alternatively, a hollow-core splice fiber (HCSF) was proposed for use as a macrobend sensor, and available data indicate that its sensitivity to a macrobend is so high that the detectable radius of curvature reached values as large as a few hundreds of millimeters, even though the loss mechanism could not be clearly explained.7 Here, we present a hollow-core splice scheme for use in macrobend measurements and discuss its characteristics both theoretically and experimentally. To more clearly understand the characteristics of the special fiber scheme, a perturbation theory for modes on a bent hollow-core fiber (HCF) and its loss characterization are presented. In the following sections, we present a brief theory of a bent HCSF and experimental and numerical results relative to its characteristics and, finally, we summarize the present status of the issue.

2 Theory of a Bent HCSF

Optical modes, restricted to the case of core modes, in HCFs are leaky, because they undergo Fresnel reflections, rather than total reflections, on the core-cladding boundary due to the fact that the refractive index of the core is smaller than that of the cladding.23-26 The propagation loss is proportional to \( \lambda^2/a^3 \), where \( \lambda \) and \( a \) are the wavelength and the radius of the hollow core, respectively.21 However, the loss can be reduced to a considerable extent when the core radius increases to values as large as hundreds of micrometers, and, because of this, they have been investigated for use in long-distance optical transmission and lasers.23,24 On the other hand, if the core radius is reduced to the level of a few micrometers, the core mode becomes too leaky to propagate even in a short length. For example, if the core radius is 4 \( \mu \)m, the propagation loss is estimated to be approximately -70 dB/mm, according to Ref. 23. In this case, if the refractive index of the surrounding region is
with decreasing radius of curvature, and this phenomenon causes a change in coupling efficiency between the fibers. 16,17 This can be applied to the measurement of a macrobend, and thus the mode characteristics can be analyzed.

According to Ref. 21, the approximated scalar wave equation for a linearly polarized mode in a bent fiber is given by

$$\left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k_n^2 + \left( 1 + \frac{r}{R \cos \phi} \right)^{-2} \right] \bar{E} = 0,$$

where $E$, $\beta$, $k$, and $n$ are the perturbed electric field, the corresponding propagation constant, the propagation constant in a vacuum, and the refractive index of the fiber, respectively; and $R$ is the radius of curvature of a bent fiber, as shown in Fig. 2. Assuming that the fiber radius of interest is much smaller than $R$, i.e., $b/R \ll 1$, where $b$ is the radius of the fiber, a perturbation theory can be used to solve Eq. (1), expanding $E$ in an ascending series of $b/R$ as

$$E = \sum_{m=0}^{\infty} (b/R)^m E_m,$$

where $E_0$ denotes the eigenmode electric field for the case of a straight fiber. Using a Taylor expansion on the last term of Eq. (1) and taking the first-order perturbation term into consideration, the following equation can be obtained 16,17:

$$\left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k_n^2 - \beta_0^2 \right] E_1$$

$$= -\frac{r}{2} \beta_0^2 \cos \phi E_0,$$

where $\beta_0$ denotes the propagation constant of the eigenmode. Thus, the mode characteristics are obtained by solving the nonhomogeneous differential equation of Eq. (3).

The fundamental eigenmode for $E_0$ in the HCF should be taken as an LP$_{01}$ mode, since the mode in the SMF is a single LP$_{01}$ mode, and the coupling between modes of the same azimuth order becomes fundamental. 18 Thus, the eigenmode field $E_0$ is given by

$$E_0 = \begin{cases} A_1 I_0(h_1 r) & 0 < r < a \\ A_2 J_0(h_2 r) + A_3 Y_0(h_2 r) & a < r < b \\ A_4 K_0(h_3 r) & b < r, \end{cases}$$

where

Fig. 1 Partial image of an HCSF fabricated by fusion splicing an HCF segment between SMFs. The core and cladding diameters of the fibers are: 8 and 110 $\mu$m for the HCF and 8 and 125 $\mu$m for the SMF, respectively. The collapsed length is approximately 220 $\mu$m.

smaller than that of the cladding. Cladding modes become fundamental modes and as a result, light is guided by the cladding rather than by the core. 22,23,24 Thus, to understand the bending effect in an HCF with a small core radius, the characteristics of the cladding modes must be analyzed.

A partial image of an HCSF is shown in Fig. 1, in which an HCF segment has been fusion-spliced between SMFs. Note that it is surrounded by air, since the jacket of the fiber has been removed. We can see that the beginning of the HCF collapses by fusion welding, and acquires a cone-like shape. Thus, it becomes difficult for the incident light from the SMF to enter the hollow-core region. In other words, it is ready for the incident light to enter the cladding region of the HCF, so that the cladding modes are excited in the HCF dominantly. Next, the cladding modes through the HCF are coupled into the core of the SMF on the other side of the connection via the collapsed region, even though there is a loss of coupling, which represents an intrinsic insertion loss. In addition, it can be stated that the collapsed region provides a path for the cladding modes through the HCF to enter the SMF on the junction point and, thus, aids the mode coupling at the fiber junction. Empirically, we can see that the coupling efficiency increases after the fusion welding of the two fibers. Note that cladding modes are highly sensitive to fiber bends, since the mode diameters are so large, compared to the case of core modes, where the mode profiles are ready to shift away from the guide axis. 13,14,15 The HCSF scheme takes advantage of this property for a macrobend measurement.

In general, the entire loss of a bent fiber consists of two dominant loss mechanisms: pure bend loss and transition loss. 4 One results from the power loss of modes in the radial direction due to the extreme bend of the fiber. The other occurs when the mode profiles between two fiber segments are not identical, such as in the case of straight and the bent fiber segments. Assuming a macrobend with a small value of bend curvature, the pure bend loss in HCF becomes negligible, since its numerical aperture for cladding modes is very large 18 ($\Delta n \approx 0.444$). Thus, the bend loss in an HCSF is largely due to the mode-transition loss, i.e., to the mode-coupling loss between the HCF and the SMF. The mode profile shifts away from the guide axis

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Fig. 2 (a) Local coordinate system of a bent fiber, in which \( R \) denotes the radius of curvature of a bent fiber and (b) HCF structure, in which each \( n_i \) denotes the refractive index of the corresponding region.

\[
\begin{align*}
    h_1^2 &= \beta_0^2 - n_1^2 k^2, \\
    h_2^2 &= n_2^2 k^2 - \beta_0^2, \\
    h_3^2 &= \beta_0^2 - n_3^2 k^2.
\end{align*}
\]

Note that the optical mode propagates through the cladding region and evanesces in its exterior, i.e., both in the core and surrounding regions. Finally, the solution of Eq. (3) is given by

\[
E_1 = \begin{cases} 
-\frac{\beta_0^2}{2b h_1} I_1(h_1 r)(A_1 r^2 + B_1) \cos \phi & 0 < r < a \\
-\frac{\beta_1^2}{2b h_2} \left[ J_1(h_2 r)(A_2 r^2 + B_2) + Y_1(h_2 r)(A_3 r^2 + B_3) \right] \cos \phi & a < r < b \\
\frac{\beta_0^2}{2b h_3} K_1(h_3 r)(A'_4 r^2 + B'_4) \cos \phi & b < r.
\end{cases}
\]

(6)

The detailed derivation and parameters are provided in the appendix. As a result, the perturbed-mode field can be expressed by

\[
E = E_0 + \frac{b}{R} E_1.
\]

(7)

From the derivations, the mode profiles of several lowest order modes are shown in Fig. 3. The parameters of an HCF are provided in the figure caption. We can see that the mode fields oscillate within the cladding region based on the mode order, which also shifts away from the guide axis with the radius of curvature.

The exact estimation of the coupling loss between the HCF and the SMF poses a very complex problem, because of the complicated structure of the junction with the fusion welding. However, if we concentrate our attention on the coupling-loss variation with respect to the radius of curvature of a bent HCSF, and regard the intrinsic coupling loss due to the geometry as implicit, the variation can be expressed by the ratio compared to the case of a straight fiber. Thus, the relative coupling efficiency \( \eta_r \) can readily be obtained by the overlap integral between the eigenmode field \( E_0 \) and the perturbed field \( E \) as

\[
\eta_r^2 = \frac{\int \int E_0^*(r, \phi) E(r, \phi) r \, dr \, d\phi}{\int \int |E_0(r, \phi)|^2 r \, dr \, d\phi \int \int |E(r, \phi)|^2 r \, dr \, d\phi}.
\]

(8)

Thus, it becomes possible to theoretically estimate the bend loss of an HCSF by using the relations through Eqs. (6) and (8). The mathematical formulas for the Bessel function integrals in Eq. (8) can be found in Ref. 30.
Within $100^\circ$C and $100 \, \mu eps$, respectively, which was nearly negligible.

Numerical results are shown in Fig. 4(b) for the bent HCSF with several lowest order modes, the parameters of the HCF are the same as described in Fig. 3. The issue of what types of modes are excited in the HCF is unclear because of the abrupt junction produced by fusion welding. However, the fundamental mode will be an LP$_{0m}$ mode or a combination of several LP$_{m}$ modes so that they can be coupled into the LP$_{0m}$ on the SMF, even though they are dependent on the detailed geometry of the junction. To evaluate the entire mode coupling between the HCF and the SMF, a mode excitation rule should be considered, for example, an overlap integral evaluation between the modes of the HCF and the SMF. The overlap integrals between the various modes (LP$_{01}$, LP$_{02}$, and LP$_{03}$) of the HCF and the fundamental mode of the SMF can be obtained. They are given by 0.060532 for LP$_{01}$, 0.10326 for LP$_{02}$, and 0.12900 for LP$_{03}$ with respect to the fundamental mode of the SMF, which was normalized such that the auto-overlap integral is unity. The values for the higher order modes are in the range of around 0.1, even though some variations are obvious. We can see that the overlap integral value for LP$_{01}$ of the HCF is smaller than the others, and thus, its probability of excitation is also small. Thus, the loss curve to bend may follow the trend for higher order modes (LP$_{02}$ and LP$_{03}$) rather than for the LP$_{01}$ mode, which can be seen in the comparison of Figs. 4(a) and 4(b). However, the geometry of the splice region is not a simple one, as shown in Fig. 1. Thus, its precise formalization exceeds the scope of this paper.

Note that numerical bend losses of the lowest order modes are in a similar range to the case of the experiments. For the higher order modes, the bend losses decrease slightly, but the trends of variation are similar, even though their characteristics are not provided for visual aids. Thus, we can see that the numerical estimation by the first-order perturbation theory for the field profile shows modal trends that are in good agreement with experimental data.

4 Conclusion

A hollow-core splice scheme was presented for use in macrobend measurement and its characteristics were discussed both theoretically and experimentally. To understand the characteristics of the special-fiber scheme clearly, a perturbation theory was presented for modes on a bent HCF and its loss characterization. HCSF sensors were fabricated for use in measuring the macrobend. The maximal detection range of the radius of curvature reached values as large as a few hundreds of millimeters. The sensitivities of the sensors to temperature and strain were determined to be within $\pm 0.05 \, \text{dB}$ in the range within $100^\circ$C and $100 \, \mu eps$, respectively. In addition, the numerical estimation by the first-order perturbation theory for the field profile verified modal trends in good agreement with experimental data. Even though the hollow-core splice scheme for a macrobend measurement has a relatively large insertion loss of approximately 5 dB, the high sensitivity to a small-curvature bend is applicable to macrobend sensors, which is the most favorable advantage of this scheme.
5 Appendix

In Eq. (4), the coefficients $A_j$'s can be found from the continuity of the electric and magnetic fields of the eigenmode at the boundaries \( r = a \) and \( r = b \) (Ref. 18) and are given by

\[
A_2 = -\frac{\pi a}{2} \left[ h_2 Y_1(h_2a) I_0(h_1a) + h_1 Y_0(h_2a) I_1(h_1a) \right] A_1,
\]

\[
A_3 = -\frac{\pi a}{2} \left[ h_2 J_1(h_2a) I_0(h_1a) + h_1 J_0(h_2a) I_1(h_1a) \right] A_1,
\]

\[
A_4 = -\frac{1}{K_0(h_2b)} [ A_2 J_0(h_2b) + A_3 Y_0(h_2b) ],
\]

\[
h_3 A_4 K_1(h_2b) = h_2 A_2 J_1(h_2b) + h_2 A_3 Y_1(h_2b),
\]

where the last represents the characteristic equation that determines the eigenmode propagation constant $\beta_0$, and the coefficient $A_1$ can be chosen arbitrarily based on the power flow of the eigenmode.

The general solution of the nonhomogeneous differential Eq. (3) can be found by the method of variation of parameters as Eq. (6), the procedure of which is lengthy but straightforward.\textsuperscript{12} The undetermined matching coefficients $B_j$'s can also be found from the continuity of the electric and magnetic fields of the perturbed mode at the boundary \( r = a \) and \( r = b \) in a matrix form:

\[
U_i B_j = V_i, \quad i, j = (1, 2, 3, 4),
\]

with

\[
U_{11} = -I_1(h_1a) / h_1,
\]

\[
U_{12} = J_1(h_2a) / h_2,
\]

\[
U_{13} = Y_1(h_2a) / h_2,
\]

\[
U_{21} = -J_1'(h_1a),
\]

\[
U_{22} = J_1'(h_2a),
\]

\[
U_{23} = Y_1'(h_2a),
\]

\[
U_{32} = J_1(h_2b) / h_2,
\]

\[
U_{33} = Y_1(h_2b) / h_2,
\]

\[
U_{34} = K_1(h_2b) / h_3,
\]

\[
U_{42} = J_1'(h_2b),
\]

\[
U_{43} = Y_1'(h_2b),
\]

\[
U_{44} = K_1'(h_2b),
\]

\[
V_2 = I_1(h_1a) a^2 A_1 / h_1 - J_1(h_2a) a^2 A_2 / h_2
\]

\[
- Y_1(h_2a) a^2 A_3 / h_2,
\]

\[
V_3 = J_1(h_2b) b^2 A_2 / h_2 - Y_1(h_2b) b^2 A_3 / h_2
\]

\[
- K_1(h_2b) b^2 A_4 / h_3,
\]

\[
V_4 = -I_1'(h_2b) b^2 A_2 / h_2 + 2J_1(h_2b) b^2 A_3 / h_2
\]

\[
- Y_1'(h_2b) b^2 A_4 / h_3
\]

where the other components not shown are zeros, and the primes denote the derivative with respect to the argument of the function.

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References


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