Residual Stresses in a Doubly Clad Fiber with Depressed Inner Cladding (DIC)

Y. Park, K. Oh, U. C. Paek, Senior Member, IEEE, Fellow, OSA, D. Y. Kim, and Charles R. Kurkjian

Abstract—Thermal and mechanical stresses developed in concentric three-layered optical fiber-core, and inner and outer cladding, have been thoroughly studied for various concentrations of dopants and geometric structures. In order to examine the parametric results of thermal stresses in preforms, the stresses were measured with a polariscope. The results agreed well with the theoretical calculations. The thermal stresses were calculated for three temperature ranges in which the glass in each layer has a different thermal expansion coefficient. The stresses were studied considering the normal stress in the molten neck down region and its development with time. In order to include the time dependence of the stress below softening point, Maxwell’s one dimensional viscoelasticity was applied. In a parametric study, the analyzes were carried out based on the fiber parameters such as relative index difference, ratio of clad to core, and depressed relative index difference. With an increase of core index above the silica, the thermal stresses in core increased linearly, but the depressed inner clad does not affect the stresses in core. From the parametric studies and modeling it was found that when the depressed inner cladding (DIC) layer has a large cross-section or high dopant concentration, the mechanical stress in core change from compression to tension.

Index Terms—Dispersion compensating fiber (DCF), fiber Bragg grating (FBG), long-period fiber grating (LPFG), measurement, mechanical stress, optical fiber, thermal stress, viscoelasticity.

I. INTRODUCTION

As the demands for communication bandwidth rapidly increase, researches on the fabrication of large preforms are being pursued actively for cost effective mass production. The core rods for large preforms in the modified chemical vapor deposition system (MCVD) require thick layers of different compositions of glass for the core and inner cladding. The overall dimensions of these layers are comparable to that of the core in multimode fibers. The composition and geometry of each layer affects the optical and mechanical properties. Recently the importance of dispersion compensating fibers (DCF) has been emphasized in order to reduce the dispersion penalties in conventional single mode fiber networks for high data rate transmission. Compared to simple conventional single mode fibers, DCF’s demand high dopant concentrations in both the core and inner cladding in order to satisfy the desired waveguide structures for negative value of dispersion. These fiber preforms, which have been developed recently, are very likely to develop high mechanically and thermally induced stresses compared with conventional small-size single mode fiber preforms due to their expanded deposition layer thickness and high concentration of dopants. It has been known empirically that residual stresses in optical fibers affect scattering loss and lead to undesirable refractive index variation. It is therefore, necessary to understand the basic physics of stress development in optical fibers and preforms with multiple layers of different glass composition in order to optimize the structure for improved optical properties as well as mechanical reliability. To evaluate residual stresses in the optical fiber preforms being presently developed, the interaction among the layers must be treated in detail because of their structural geometry and dopant concentration.

The recent development of fiber Bragg gratings (FBG’s), as well as long-period fiber gratings (LPFG’s) has also led to a need to understand the development as well as the relaxation of thermally and mechanically induced stresses in fibers. In particular in those FBG’s and LPFG’s in which there is a substantial compaction in the formation of the grating, or in those cases where a mechanically induced stress is relaxed to form the grating, these processes are important [1]–[3].

In 1970, the simplest analysis of a graded-index fiber was given by Timoshenko [4]. Subsequently Brugger [5] explained that significant changes in index would occur only if the expansion difference between core and cladding were much greater than the index difference. Moreover, Krohn and Cooper [6], [7] pointed out that if the core is more fluid than the cladding, a hydrostatic stress appears in the core as soon as the cladding sets. Scherer and Cooper [8] treated the stress development in a step-index fiber based on the simple assumption that the glass behaves like a Maxwell element in shear.

An additional residual stress can be mechanically induced by the force used to draw a fiber. It is related to the cross sectional areas, the elastic moduli and the viscosities of each of the fiber layers [9], [10]. Paek and Kurkjian [9] have calculated the mechanically induced stresses for step index fibers, pointing out that the viscosity difference between core and cladding causes the initial stresses to depend on the pulling load, and that only the elastic strain parts of the initial strains can be recovered to mechanical equilibrium position after the load is removed. In 1989, Hibino et al. [10], [11] solved
the stress responses for index variation by considering this viscosity effect at the end of neck-down region.

The residual stress in the drawn fiber is a linear superposition of thermal and mechanical stresses. Many researchers have measured the residual stresses in preforms and fibers. The measurement of stresses in an optical fiber was investigated for five different fibers by Saunders [12]. Shibata [13] studied the relation between the optical retardation of the ray transmitted through the preform and its structure information by using a nondestructive measurement based on the photoelastic effect. Chu and Whitbread [14] suggested that only the axial stress component can contribute to the phase retardation of two linear polarization components, the Abel integral transformation of which is the axial stress distribution in the radial direction. It was further shown that the thermal stress profiles of the preform and the fiber drawn from it are not much different if the mechanically induced contribution is small.

In this study, the residual stresses in optical fibers and preforms of the three layered structure which are denoted by the core, the inner cladding and the outer cladding, have been analyzed theoretically. Especially the thermally and mechanically induced stresses of a three layered preform, GeO2-SiO2 core, F-SiO2 inner cladding and cladding, have been studied for the first time. The thermal stresses in doubly clad fiber structures have been calculated for three temperature ranges defined by individual glass transition temperatures of the three layers. To examine the results of the thermal stresses, measurements of the stresses in preforms have been carried out. For the mechanically induced stress, the equations applicable to the three layered structures have been constructed and the study of the time dependent viscoelastic behavior before a fiber is cooled to the elastic state have been carried out for the fiber drawing process. Additionally, the variations of the residual stresses have been studied with different dopant concentrations as well as with changes in the core-cladding ratio.

II. THERMAL STRESS

The viscosity-temperature behavior of the three layers normally used can be regarded as that characteristic of strong liquids, and therefore their temperature dependence will be governed by Arrhenius equations [15]. Moreover, for silica glasses doped with GeO2 or F, it is known empirically that the logarithmic change of viscosity depends linearly on dopant concentration with a constant activation energy [16]–[18]. These effects can be applied to the Arrhenius equation by simply appending the terms which depend on the relative index difference, \( \Delta \). The value of \( \Delta \) in a fiber is defined by \( \Delta = (n_1 - n_F)/n_1 \). Similarly, \( \Delta^\pm \) is defined by \( (n_1 - n_2)/n_1 \) and \( \Delta^- \) by \( (n_1 - n_2)/n_1 \). Hence, \( n_1 \) are the index of core, \( n_2 \) the index of outer cladding (silica) and \( n_1 \) the index of depressed cladding (e.g. F-doped silica). Therefore, the following relation is held, namely, \( \Delta = \Delta^+ = \Delta^- \). The viscosity of a single dopant concentration in silica glass can be expressed by

\[
\log \eta = K_0 + K_F \Delta_F \\
\log \eta = K_0 + K_{GeO_2} \Delta_{GeO_2}
\]

where \( K_0 \) is the logarithm of the viscosity for pure silica glass, \( K_{F,GeO_2} \) are viscosity sensitivity of the dopants and \( \Delta_{F,GeO_2} \) are relative index difference relative to pure silica glass. For conventional three layered single mode fibers, the viscosity of each fiber layer can be sequenced as

\[
\eta_{GeO_2} > \eta_{GeO_2} > \eta_F
\]

where \( \eta_{GeO_2} \) is the viscosity of a pure silica glass, \( \eta_{GeO_2} \) of GeO2 doped silica glass and \( \eta_F \) of F doped silica glass at a certain high temperature. For a glass which has been cooled at a normal rate, the glass transition occurs at a viscosity of about \( 10^{12} \text{ Pa} \cdot \text{s} \) \( (T \sim 1100–1200^\circ \text{C}) \), since this corresponds to a relaxation time of 100s of seconds. Thus in the case of the formation of a preform, \( T_g \) may be considered to occur at such a viscosity. Since experimental measurements of stress have been carried out only on preforms, this “normal” \( T_g \) will be used for these calculations. On the other hand, as shown by Paek and Kurkjian, 125 \( \mu \text{m} \) silica fibers cool much more rapidly. This causes the freezing of the glass structure (or the ‘apparent’ glass transition) to occur at substantially higher temperatures. The estimation by Hibino, et al. [19] indicates that this temperature may be as high as 1800–2000\(^\circ\)C. For calculations on fibers, such an elevated \( T_g \) must be used. The information above on the viscosity permits us to estimate the glass transition temperature at which the expansion coefficients change sharply from that of a liquid-like expansion coefficient, which is approximately three times that of the solid glass. To theoretically calculate the thermal stresses, the glasses at temperatures above \( T_g \) are assumed to be fluid, and thus the hydrostatic stress is exerted on the other solid layers. In calculating the thermal stresses, the temperature ranges must be divided because each layer has a different glass transition temperature. In Fig. 1, for example, when the temperature of the preform is between \( T_{g_3} \) and \( T_{g_1} \), the outer cladding is almost solidified, while other two layers are still in fluid state. This means that the stress starts to develop at \( T_{g_3} \). Therefore, the thermal stresses over these three temperature ranges are calculated utilizing different thermal expansion coefficients \( (\alpha_1, \alpha_2, \alpha_3, \alpha_3^+, \alpha_3^-) \). The \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) values in region of \( T < T_{g_2} \) are \( 7.5 \times 10^{-7} \text{ \(^\circ\)C}^{-1}, 4.3 \times 10^{-7} \text{ \(^\circ\)C}^{-1} \) and \( 4.6 \times 10^{-7} \text{ \(^\circ\)C}^{-1} \), respectively. \( \alpha_{1} = 7.5 \times 10^{-7} \text{ \(^\circ\)C}^{-1}, \alpha_{2} = 13 \times 10^{-7} \text{ \(^\circ\)C}^{-1} \) and \( \alpha_{3} = 4.3 \times 10^{-7} \text{ \(^\circ\)C}^{-1} \) for \( T_{g_2} < T < T_{g_1} \). For \( T_{g_1} < T < T_{g_3} \), \( \alpha_{1}^{+}, \alpha_{2}^{+} \) and \( \alpha_{3} \) are \( 22.5 \times 10^{-7} \text{\(^\circ\)C}^{-1}, 13 \times 10^{-7} \text{\(^\circ\)C}^{-1} \) and \( 4.6 \times 10^{-7} \text{\(^\circ\)C}^{-1} \), respectively.

In the temperature range from \( T_{g_3} \) to \( T_{g_1} \), the two inner layers are in the fluid state while the outer cladding layer is in the solid state. The three components of the stress should be equal under the assumption of the hydrostatic pressure

\[
\sigma_{r_1,2} = \sigma_{\theta_1,2} = \sigma_{z_1,2} = \sigma_{r_1,2}
\]

where \( \sigma_{r_1,2} \) is the stress in the radial direction in region 1 (core) and 2 (inner clad). The same is true for the stress in the \( \theta \) and \( z \) directions. The outer cladding with cylindrical geometry is known to have the components of the stress [20]

\[
\sigma_{r_3} = A_3 + \frac{B_3}{r^2}
\]
Fig. 1. The temperature ranges divided by the glass transition temperatures. Note that in the liquid state \( \alpha^* \) is assumed to be three times larger than that in solid.

Given by

\[
\sigma_{r1} = A_1 \quad (9) \\
\sigma_{\theta 1} = A_1 \quad (10) \\
\sigma_{z1} = C_1. \quad (11)
\]

Also the boundary conditions in the second temperature range are shown in the Table I. They are very similar to those in the first temperature range with the addition of the condition that the radial stress and displacement and axial strain must be continuous at the interface between the core and inner cladding.

Finally, as seen in Fig. 1, when the temperature decreases below \( T_{g2} \), the inner cladding changes from the liquid state into the solid state. The thermal stress for this case has been calculated by Timoshenko [4]. In contrast to \( \Delta T \) as an \( r \)-dependent variable in his analysis, our analysis is based upon the assumption that the thermal expansion coefficient depends on \( \tau \), rather than \( \Delta T \). Then the thermal stress in each direction can be rearranged as

\[
\sigma_r(r) = -\frac{1}{r^2} \frac{E \Delta T}{1-\nu} \int_0^r \alpha(r') r' dr' + C_1 \quad (12)
\]

\[
\sigma_\theta(r) = \frac{1}{r^2} \frac{E \Delta T}{1-\nu} \int_0^r \alpha(r') r' dr' + C_1 - \frac{\alpha(r) E \Delta T}{1-\nu} \quad (13)
\]

\[
\sigma_z(r) = 2 \tau C_1 - \frac{\alpha(r) E \Delta T}{1-\nu}. \quad (14)
\]

Note that only one unknown constant \( C_1 \) exists in the above solutions. From the boundary condition of the radial stress, the unknown constant \( C_4 \) can be written as

\[
C_4 = \frac{1}{2} \frac{E \Delta T}{1-\nu} \int_0^r \alpha(r') r' dr'. \quad (15)
\]
Fig. 2. Total thermal stress profile of F-doped DIC fiber. In this calculation, the elastic modulus $E = 7.0 \times 10^9$ Pa, bulk modulus $K = 4.12 \times 10^9$ Pa, $T_{g1} = 1139.8 ^\circ C$, $T_{g2} = 1130.1 ^\circ C$, $T_{c1} = 1058.6 ^\circ C$, core radius $a = 4 \mu m$, outer radius of inner clad $b = 20 \mu m$, outer radius of outer clad $c = 62.5 \mu m$, $\alpha_1 = 7.51 \times 10^{-7}$, $\alpha_2 = 4.347 \times 10^{-7}$ and $\alpha_3 = 4.6 \times 10^{-7}$.

The total thermal stress is the sum of the thermal stresses in each temperature range, which is shown in Fig. 2.

To study the behavior of the DIC layer, we consider the stress variation with the dopant concentration and the ratio, $D/d$ of the inner clad diameter $D$ to the core diameter $d$. In order for the fiber to support a single mode, we fixed the core radius as $4 \mu m$ and the relative index difference $\Delta$ as 0.0035. For a given $\Delta$ and $\Delta^-$, the glass transition temperatures can be obtained from (1) and (2) when they occur at viscosity about $10^{12}$ Pa·s. The thermal expansion coefficient is a linear function of the dopant concentration for a lightly doped silica glass. Figs. 3 and 4 show the stress variation with $D/d$ and with $\Delta^-$, respectively. Note that the relative index difference $\Delta$ is the difference of $\Delta^+$ (core) and $\Delta^-$ (clad). The first and the second temperature ranges are expected to be large since the stress begins to develop in the case of the more...
Fig. 4. The variation in thermally induced stresses in the core region with $\Delta^+$. Note that the total relative index $\Delta = \Delta^+ - \Delta^-$ is kept constant. Here the core radius $a = 4 \mu m$, outer radius of outer clad $c = 62.5 \mu m$, $\Delta = 0.0035$, $D/d = 7$. To obtain the glass transition temperatures and viscosities, $\log \eta_0 = 5.5$, activation energy $E = 117$ kcal/mole, and gas constant $R = 0.001987$ kcal/K \cdot mole. The viscosity sensitivities of GeO$_2$ and F-doped silica glass, $R_{GeO_2} = 0.5$ and $R_F = 1.5$.

viscous glass, rather than the less viscous one. However in this calculation, only a small amount of F is contained in the inner cladding layer. Therefore, $T_{g2}$ is not much different from $T_{g3}$ and $T_{g1}$. The results of the calculation in the first and the second temperature ranges ($T_{g3} - T_{g1}, T_{g1} - T_{g2}$) may be much smaller than that in the third temperature range ($T_{g2} - T_o$). In Fig. 3, when $D/d$ is increased, the stresses in the core region increase slowly but those increasing ranges are smaller than 1 MPa. This means that the effect in the core region of the DIC may be negligible. And also in Fig. 4 when $\Delta^+$ is increased, the stresses in core region increase linearly. Hence, in Fig. 4, it can be found that the DIC doesn’t affect the stresses in core region. The linear decrement of thermal stresses in the core may result from the decrement of the difference of the thermal expansion coefficient between core and outer cladding by decreasing the $\Delta^+$ in the third temperature range.

III. MECHANICAL STRESS

It is known that viscosity differences result in stresses being mechanically induced during the fiber drawing process [9]. When a pulling force is applied, a large axial stress relative to the other stresses can be expected because the axially induced deformation length is much greater than that in the transverse direction. Thus the mechanically induced stress can be considered as a one dimensional viscoelastic problem. In this analysis there are two conditions denoted as the initial and final condition. The initial condition is under load at high temperature but does not contain a time dependent response which results from the viscoelastic behavior of a glass. Therefore, the initial condition will be set after the time dependent process of each fiber layer is finished. In the final condition, the fiber is cooled to room temperature and the load removed. Fig. 5 shows a schematic of the initial and final conditions of DIC fibers in the drawing process. Until the load is removed, the deformations induced from the initial condition are stationary and the ratio between the elastic and

Fig. 5. A schematic of the initial and final conditions. Schematic (a) shows the induction of tensile stress in the core for heavily doped inner clad and (b) the induction of compressive stress in the core for lightly doped inner clad.
in elastic modulus is assumed to be small. From the sequences of the initial viscosity of each layer in (3), the initial stresses can be ordered as

$$\sigma_3^i > \sigma_1^i > \sigma_2^i. \quad (16)$$

After the fiber passes through a capstan, the deformed fiber layers tend to recover instantaneously. However the elastic strains are different and some parts of the inelastic deformation may be stressed again elastically satisfying the mechanical equilibrium in the axial direction.

Now we can consider the two cases of the mechanically induced residual stresses in an F-doped DIC fiber in Fig. 5. In one case, a tensile stress is induced in the core layer; Fig. 5(a). In the other case, compressive stress is induced in the core layer; Fig. 5(b). When the DIC is doped highly enough, the viscosity differences may be substantial at high temperature. In this case the initial stress in the inner clad becomes very small. When the DIC has a large cross-sectional area, the inelastic parts of the inner cladding may prevent the elastic parts of the core from recovering to the inelastic level. After removing the load, the recovery process in the core region is not perfect. Then a tensile stress may occur in the core region.

On the contrary, when the dopant concentration in the inner clad is low, the initial stress in inner cladding is nearly same as that in the other layers. When the DIC has small cross-sectional area, the elastic parts of the inner clad is recovered substantially. Then, after removing the load, the recovery process in the core region is excessive as seen in Fig. 5(b). A compressive stress may occur in core.

**A. Initial Stress: Normal Stress of Molten Neck-Down**

In 1987 [11], Hibino et al. determined the initial conditions at a temperature below the softening temperature by applying the relationship between the tension \( F \) and the normal stress of the molten neck-down region in the preform. The relation between the pulling tension \( F \) and the normal stress of the end of the neck-down region can be expressed as [21]

$$F = 3\eta_1 A_1 \frac{\partial \nu}{\partial x} + 3\eta_2 A_2 \frac{\partial \nu}{\partial x} + 3\eta_3 A_3 \frac{\partial \nu}{\partial x} \quad (17)$$

where \( A_1 \) is the cross-sectional area of core layer, \( A_2 \) of inner cladding, \( A_3 \) of outer clad, and \( \nu \) is the velocity as a function of the axial coordinate only.

The initial stresses can be written as

$$\sigma_1^i = 3\eta_1 \frac{\partial \nu}{\partial x} = \frac{F}{A_1} G_1 \quad (18)$$

$$\sigma_2^i = 3\eta_2 \frac{\partial \nu}{\partial x} = \frac{F}{A_2} G_2 \quad (19)$$

$$\sigma_3^i = 3\eta_3 \frac{\partial \nu}{\partial x} = \frac{F}{A_3} G_3 \quad (20)$$

where

$$G_1^{-1} = 1 + \frac{\eta_2 A_2}{\eta_1 A_1} + \frac{\eta_3 A_3}{\eta_1 A_1} \quad (21)$$

$$G_2^{-1} = 1 + \frac{\eta_1 A_1}{\eta_2 A_2} + \frac{\eta_3 A_3}{\eta_2 A_2} \quad (22)$$

$$G_3^{-1} = 1 + \frac{\eta_2 A_2}{\eta_3 A_3} + \frac{\eta_1 A_1}{\eta_3 A_3} \quad (23)$$

These initial stresses are applied at about the softening temperature. Therefore the initial position of the one dimensional problem may be defined at the end of the neck-down shape where the glass may be cooled down to the apparent softening temperature.

But below the softening temperature the glasses still have viscous properties (viscoelasticity) until the glasses are cooled to the normal glass transition temperature. Therefore, the deformations develop slowly to glass transition temperatures. In order to include this time dependency in the calculation, we applied Maxwell’s one dimensional viscoelasticity [22] to calculate the development of elastic strains below the softening temperature of silica glass.

**B. Initial Stress: Stress Development Using Maxwell Model**

The DIC fibers consist of composite cylinders having layers of three different dopant concentrations. Their axial strains must be continuous along the \( r \) direction since the interface remains intact and the total axial deformation which contains the elastic and the inelastic deformation both must be the same. It is reasonable to assume that each layer is a one dimensional linear viscoelastic component and the components are connected in a parallel configuration. In addition to the above assumption, we know that from Fig. 5, mechanically induced residual stress in fiber can be treated by modeling the two conditions of the drawing process with separate strains (elastic, inelastic), the superposition of which is expected to be the total strain. Therefore, the linear viscoelastic elements can be assumed to be a Maxwell element which has the following pertinent consideration. For a single Maxwell element, complete removal of pulling force is followed immediately by recovery of elastic strain but still a permanent strain (inelastic strain) remains. In the set of elements with a parallel configuration, the recovery of elastic strain for a element may cause shrinking or expanding of the inelastic strains in other elements which becomes an elastic strain at the final condition. Therefore, that configuration may be consistent with the picture for the process of a mechanically induced stress as in Fig. 5.
Fig. 6. The development of initial stresses with time from the softening point of pure silica glass, \((T_s = T_r)\), at \(t = 0\). Here, the specific heat \(C_p = 0.25\) cal/g°C, diameter \(d = 125\) µm, density \(\rho = 2.2\) g/cm³, heat transfer coefficient \(h = 7.2 \times 10^{-7}\) cal/cm²°C, core radius \(a = 4\) µm, outer radius of inner clad \(b = 20\) µm, and \(F = 30\) g.

or stress. Therefore, elastic strain \(\varepsilon^e\) and inelastic strain \(\varepsilon^v\) may be written as

\[
\sigma = \rho \varepsilon^v
\]

\[
\sigma = E \varepsilon^e
\]

(24)

(25)

where \(\sigma\) is the pulling stress. In the Maxwell model, the total strain is the sum of the elastic strain and viscous (inelastic) strain. Then the total strain is written as

\[
\varepsilon(t) = \frac{\sigma(t)}{E} + \int_0^t \frac{\sigma(t)}{\eta(t)} \, dt.
\]

(26)

As the fiber is drawn, the fiber is cooled down continuously. If the draw speed is fixed, the viscosity can be expressed as a function of time since the viscosity is a function of temperature. The expression of the total stress and the strain can be written as

\[
\varepsilon_1(t) = \varepsilon_2(t) = \varepsilon_3(t)
\]

\[
\varepsilon_0 = \varepsilon_1(t) + \varepsilon_2(t) + \varepsilon_3(t).
\]

(27)

(28)

where \(\varepsilon_0\) is defined from the sum of the axial stress of molten neck-down

\[
\sigma_0 = \left(\frac{G_1}{A_1} + \frac{G_2}{A_2} + \frac{G_3}{A_3}\right)E
\]

\[
= A_1 \eta_1(T_s) + A_2 \eta_2(T_s) + A_3 \eta_3(T_s).
\]

(29)

(30)

From (26) the strain elements of each layer can be given as

\[
\varepsilon_1(t) = \frac{\sigma_1(t)}{E} + \int_0^t \frac{\sigma_1(t)}{\eta_1(t)} \, dt
\]

\[
\varepsilon_2(t) = \frac{\sigma_2(t)}{E} + \int_0^t \frac{\sigma_2(t)}{\eta_2(t)} \, dt
\]

\[
\varepsilon_3(t) = \frac{\sigma_3(t)}{E} + \int_0^t \frac{\sigma_3(t)}{\eta_3(t)} \, dt.
\]

(31)

(32)

(33)

From (27) and (31)–(33), we can obtain the two independent linear equations

\[
\frac{\sigma_1}{E_1} - \frac{\sigma_2}{E_2} + \int_0^t \frac{\sigma_1(t)}{\eta_1(t)} - \frac{\sigma_2(t)}{\eta_2(t)} \, dt = 0
\]

\[
\frac{\sigma_1}{E_1} - \frac{\sigma_3}{E_3} + \int_0^t \frac{\sigma_1(t)}{\eta_1(t)} - \frac{\sigma_3(t)}{\eta_3(t)} \, dt = 0.
\]

(34)

(35)

Using (28) and (34) and (35), we can obtain simultaneous ordinary differential equations as follows:

\[
\frac{d\sigma_1}{dt} = \frac{E}{3} \left\{ \left(\frac{2}{\eta_1} + \frac{1}{\eta_3}\right)\sigma_1 - \left(\frac{1}{\eta_3} - \frac{1}{\eta_2}\right)\sigma_2 - \frac{\sigma_0}{\eta_3} \right\}
\]

\[
\frac{d\sigma_2}{dt} = \frac{E}{3} \left\{ \left(\frac{1}{\eta_1} - \frac{1}{\eta_3}\right)\sigma_1 + \left(\frac{1}{\eta_3} - \frac{4}{\eta_2}\right)\sigma_2 + \frac{\sigma_0}{\eta_3} \right\}.
\]

(36)

(37)

The viscosity is given by (1) and (2) and the cooling rate upon fiber drawing can be expressed by the following: [15]

\[
\frac{T - T_0}{T_s - T_0} = \exp\left(-\frac{Ah}{\rho C_p \eta d}\right)
\]

(38)

where \(\rho, C_p\) and \(d\) are the density, specific heat of the fiber material, and fiber diameter, respectively. \(T_0\) can be regarded as room temperature and \(T_s\) as the softening temperature of pure silica glass (1600 °C). We calculated the initial stress with (36)–(37) and (1) and (2) using fourth order Runge-Kutta method. The initial values of \(\sigma_1, \sigma_2,\) and \(\sigma_3\) were used with (18)–(20) at the softening temperature of pure silica glass. Fig. 6 shows the development of the stresses from the softening point. Hence, although the fiber is cooled to the softening point, it is found that the elastic deformation of each layers can still be increasing to certain levels.
Fig. 7. Residual mechanically induced stress profiles: The pulling force on drawing process is set at 50g, where g is 9.8 N and Δ is fixed at 0.003. The solid line shows the stress profile with $D/d = 3$ and $\Delta \tau = -0.001$, dotted line with $D/d = 8$ and $\Delta \tau = -0.001$, dashed line with $D/d = 3$ and $\Delta \tau = -0.0015$. $E_{1,2,3}$ were assumed to be equal to that of pure silica glass.

When the fiber passes through a take-up reel, the stretched fiber recovers instantaneously. However, for this process two principles must be postulated. The discrepancy of the total strains between the initial and final condition must be the same in each fiber layer as seen in Fig. 5 because the interfaces are intact and the axial strains at its boundary must therefore be continuous. The inelastic strain in each layer however cannot recover. Therefore the discrepancy can be expressed by that of only elastic strains. In Fig. 5, when instantaneous recovery occurs, the difference between the total initial strain $\varepsilon^i$ and final strain $\varepsilon^f$ should be same as the following:

$$\varepsilon^f - \varepsilon^i = \varepsilon^f_1 - \varepsilon^i_1 = \varepsilon^f_2 - \varepsilon^i_2 = \varepsilon^f_3 - \varepsilon^i_3. \quad (39)$$

Second, at the final condition, the net force in the axial direction must be zero if no external force is applied to the fiber axially. Then, the mechanical equilibrium can be adjusted to the each layer

$$A_1 E_1 \varepsilon^f_1 + A_2 E_2 \varepsilon^f_2 + A_3 E_3 \varepsilon^f_3 = 0. \quad (40)$$

From these equations, the final strains which will be the mechanically induced residual stresses, can be derived as,

$$\varepsilon^f_1 = \frac{(A_2 E_2 + A_3 E_3) \varepsilon^i_1 - A_2 E_2 \varepsilon^i_2 - A_3 E_3 \varepsilon^i_3}{A_1 E_1 + A_2 E_2 + A_3 E_3} \quad (41)$$

$$\varepsilon^f_2 = \frac{(A_2 E_2 + A_3 E_3) \varepsilon^i_1 - A_2 E_2 \varepsilon^i_2 - A_3 E_3 \varepsilon^i_3 + \varepsilon^f_2 - \varepsilon^i_2}{A_1 E_1 + A_2 E_2 + A_3 E_3} \quad (42)$$

The initial stresses are already given in Sections III-A and -B and those may be expressed by multiplying the initial strains by elastic modulus $E_1$, $E_2$, $E_3$, respectively.

As an example, when the initial stresses are applied as the normal stress of the molten neck-down, the final stresses or residual mechanical stresses can be written as

$$\sigma_1 = \frac{F}{A_1} g \quad (44)$$

$$\sigma_2 = F E_2 \left( \frac{g - G_1}{A_1 E_1} + \frac{G_2}{A_2 E_2} \right) \quad (45)$$

$$\sigma_3 = F E_3 \left( \frac{g - G_1}{A_1 E_1} + \frac{G_3}{A_3 E_3} \right) \quad (46)$$

where

$$g = \frac{A_3 E_3 G_1 + A_2 E_2 G_1 - A_1 E_1 (G_2 + G_3)}{A_1 E_1 + A_2 E_2 + A_3 E_3}. \quad (47)$$

In Fig. 7, the profiles of residual mechanical stress with radius is shown for various index structures using (44)–(47).

Assuming that the elastic moduli of the layers are same as that of pure silica, the variation of the axial stresses in core region with $D/d$ and $\Delta \tau$ can be as shown in Figs. 8 and 9. The solid and dotted lines are the stress variations of mechanical stresses with $D/d$ and $\Delta \tau$ using the initial stresses calculated by the normal stress of the molten neck-down region.
Fig. 8. Parametric study of the mechanically induced stress with $D/d$ in an F-doped inner clad fiber. The solid and dotted lines represent the initial stresses using the axial stress of the molten neck-down and stress development by Maxwell’s model, respectively. Here $\Delta = 0.0035$, $\Delta^- = -0.002$, core radius $\alpha = 4 \mu m$, radius of the fiber $c = 62.5 \mu m$, drawing force $F = 50$ g. Activation energy of the layers is 117 kcal/mole, and the other constants ($K, E, v, R, p, C_p, h$) were used with the same as given in Fig. 6.

Fig. 9. Parametric study of the mechanically induced stress with $\Delta^-$ in an F-doped inner clad fiber. In these calculations, $\Delta = 0.0035$, core radius $\alpha = 4 \mu m$, radius of the fiber $c = 62.5 \mu m$, $D/d = 5$ and the given constants are the same as given in Fig. 8.

and its stress development with time based on Maxwell’s model, respectively. The discrepancy between these two lines may be regarded as the decrement of initial stress in the core due to the viscoelastic behavior below the softening points as shown in Fig. 6. When the DIC has large cross-sectional area, the inelastic part of the inner clad layer may prevent the elastic part of the core layer from recovering to inelastic level. Therefore, after removing the load, the recovery process in the core region is not perfect. Then, there will be a tensile stress in the core region. When the dopant concentration of the F-doped layer becomes high, the stress in the core also changes from compression to tension.

IV. MEASUREMENT OF THE STRESS PROFILES
The thermally induced stresses in a fiber preform are very similar to those in a fiber because the processes developed
from the melting temperature on collapse may be same as fiber drawing. If the radial temperature difference in the preform does not need to be considered on collapse, measuring the stress profile in the fiber preform resulting from thermal expansion differences can be expected to be similar to that in the fiber.

In this study several preforms including F-doped DIC were made and the stress profiles of the preforms were measured using a polariscope with $\lambda/4$ wave plate similar to that described by Bachmann et al. [23], [24]. The light source is a randomly polarized 5 mW He-Ne laser. In front of the source, a linear polarizer is adjusted to 45° relative to the axial direction of the preform. The linearly polarized beam is focused on the preform axis. We used a convex lens having a 100 mm focal length. When the beam diameter of the laser is 0.8 mm, its beam waist on the preform is about 46 µm and located about 105 mm from the focusing lens. Most of the light is collected in the desired position as a finite plane wave with about 50 µm width. To minimize beam deviation, the preform is immersed in index-matching oil ($n = 1.458$) in a glass cell. A quarter wave plate is adjusted parallel to the polarization axis of the first linear polarizer. The analyzer is placed between the quarter wave plate and the detector.

The angle of the analyzer required to minimize the beam intensity was measured when the focused beam was passed through the preform transversely. The preform with an F-doped inner cladding layer was made by the modified chemical vapor deposition (MCVD) method. The substrate was synthetic tubing made by flame hydrolysis deposition of SiCl$_4$ and sintered with chlorine gas [25]. The core diameter is 1.33 mm, the $D/d$ 3.6, and the outer diameter 18.5 mm. The relative index difference $\Delta$ is about 0.34%. $\Delta$ is difference between $\Delta^-$ and $\Delta^+$, $\Delta^+$ is about 0.23%. Fig. 10 shows the refractive index profile, the retardation angle, and the axial stress profile. The refractive index profile was measured by a P104 preform analyzer (York Technology, UK). The angle profile was then transformed to a stress profile by the Abel integral transformation [14]. As seen from Fig. 10, the stress profile has very similar shape to the refractive index profile.

To examine the parametric studies of the thermal stress in an F-doped DIC fiber in Fig. 4, we made three preforms with different $\Delta^+$. The retardation angles were measured with the same experimental set up. In Fig. 11, the axial stress variations in the core were plotted with the relative index difference between the maximum core index and outer clad index. From Fig. 11, it can be seen that the experimental results agree well with the theoretical results in Fig. 4. According to parametric studies with $\Delta^+$, the axial stress in core increased linearly due to the increment of thermal expansion coefficient in core regardless of the variation of the properties in inner clad region.

**V. Conclusion**

The analytic calculation and the measurement of residual stresses have been done for the investigation of concentric three layered structure of a preform—core, inner, and outer cladding. For a given $\Delta$ and core radius, parametric analyzes were carried out for various values of $D/d$ and $\Delta^-$. With an increase in $\Delta^+$, the stresses in the core increased linearly. This statement has an important implication: the depressed inner clad does not affect the stresses in the core. The mechanically induced stresses in a fiber were studied considering the initial draw stress and its development with time. In order to include the time dependency of the stress below softening point, Maxwell’s one dimensional viscoelasticity was applied. From the parametric studies and modeling, when DIC layer has a large cross-section or high dopant concentration, the stress in the core changes from compression to tension. To examine the parametric results of the thermally induced stress in F-doped
DIC preforms, the stresses were measured with polariscope, the result of which agreed well with the theoretical results. Because of the short path lengths in the fiber samples, stress (birefringence) measurements were made only in preforms. Mechanically induced stresses were therefore not measured experimentally.

ACKNOWLEDGMENT

The authors would like to thank Korea Telecom Access Network Laboratory for providing the preform index measurement.

REFERENCES


Y. Park, photograph and biography not available at the time of publication.

K. Oh, photograph and biography not available at the time of publication.

U. C. Paek (M’90–SM’94) was born in Korea. He received the B.S. degree from Korea Merchant Marine Academy, Pusan, Korea, in 1957. He received the M.S. and Ph.D. degrees from the University of California, Berkeley, in 1965 and 1969, respectively.

From 1969 to 1991, he was with Bell Laboratories, Lucent Technologies (previously AT&T), where he was a Member of the Technical Staff, a Distinguished Member of the Technical Staff, and a Bell Labs Fellow. In 1991, he returned to Korea and became the Executive Vice President of Korea Academy of Industrial Technology. He is now a Professor of the Information and Communications Department, and the Director of the Research Center for Ultrafast Fiber-Optic Networks, Kwangju Institute of Science and Technology, Kwangju, Korea. His research interests are in the areas of optical communications, optical fiber technology, and fabrication of optical devices and components.

Dr. Paek is a Fellow of the Optical Society of America (OSA), a Fellow of American Ceramic Society, and a member of Sigma Xi.

D. Y. Kim, photograph and biography not available at the time of publication.

Charles R. Kurkjian received the B.Sc. degree from Rutgers University, NJ, in 1952 and the Sc.D. degree from the Massachusetts Institute of Technology (M.I.T.), Cambridge, in 1955.

In 1994, he retired as a Distinguished Member of Technical Staff from AT&T Bell Laboratories, Holmdel, NJ, after 35 years of service. Since that time, he has been with Bellcore, NJ, in the Fiber Media and Components Group. His principle interest is in oxide glasses, especially their mechanical properties.